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TESTING FOR FIRST ORDER SERIAL CORRELATION IN TEMPORALLY AGGREGATED REGRESSION MODELS

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ABSTRACT

This paper shows that the LM statistic for testing first order serial correlation in regression models can be computed using the Kalman Filter.

It is shown that when there are missing observations, the LM statistic for this test is equivalent to the test statistic derived by Robinson (1985) using the likelihood conditional on the observation times.

The Kalman Filter approach is preferable because the test statistic for first order serial correlation in temporally aggregated regression models can be obtained as an extension of the previous case.

KEYWORDS: First Order Serial Correlation; LM test; Regression Models; Missing Observations; Temporal Aggregation; Kalman Filter.

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1 INTRODUCTION

The most common test against autocorrelated errors in regression models is the bounds test of Durbin & Watson (1950, 1951, 1971). Many authors - Breusch (1978) and Godfrey (1978) among others- observed that the Durbin's h statistic (see Durbin (1970)) can be derived from the general theory of the LM test procedure.

When observations are missing in regression models, a modified Durbin - Watson statistic was proposed by Savin & White (1978) d Dufour & Dagenais (1985).

In a recent paper Robinson (1985) applied the LM principle and derived two test statistics, where one is obtained using an unconditional form of the likelihood and the other using the likelihood conditional on the observation times. Asymptotic distributions of the test statistics are established and analytic comparison of efficiencies are made.

The motivation for using the LM approach is that it produces an asymptotically locally most powerful test and the χ^2 asymptotic approximation to its distribution is robust to departures from normality.

This paper follows Robinson's approach and a LM statistic for testing serial correlation in regression models with missing observations is obtained. This test statistic is obtained as a by - product of the Kalman Filter recursions. It is shown that a reparametrization of the parameter space is useful in obtainig this test statistic.

This approach is used because the extention to temporally aggregateds regression models can be done very easily.

The structure of this paper is as follows. In section 2, the LM statistic for testing serial correlation in regression models with missing observations is obtained.

Section 3 extends the results to temporally aggregated regression models.

Concluding remarks are made in Section 4.

2 THE LM STATISTIC FOR TESTING FIRST ORDER SERIAL CORRELATION IN REGRESSION MODELS WITH MISSING OBSERVATIONS

Consider the regression model with autocorrelated errors, i. e.:

$$y(t) = x'(t)\beta + u(t) \quad t = 1, \ldots, T \tag{2.1}$$

$$u(t) = \rho u(t-1) + \epsilon(t) \tag{2.2}$$

where x(t) is a k×1 vector of nonstochastic regressors, independent of u(t), β is a k×1 vector of unknown parameters and $\epsilon(t)$ is i.i.d. N(0 , σ^2).

It is assumed that y(t) is observed each m time periods (sometimes called "skip sampling"), i. e. y_τ = y(mt) $\tau = 1, \ldots, [\frac{T}{m}]$ is the observed endogenous variable and x(t) is observed for each time period.

The proposition below proves that the regression model for the observed data has parameters β, $\rho_\star = \rho^m$ and $\sigma_\star = (1 + \rho^2 + \ldots + \rho^{2(m-1)})\sigma^2$. Therefore the log likelihood function for this regression model will be a function of the new parameter set $\Psi = (\rho_\star, \beta', \sigma^2_\star)$ and the LM statistic for testing $H_0 : \rho = 0$ vs $H_a : \rho \neq 0$ will be obatined using this reparametrization, because testing $\rho = 0$ is equivalent to test $\rho_\star = 0$.

Proposition I.

Consider the regression model with autocorrelated errors given by (2.1-2). If the endonegous variable y(t) is observed only for $t \equiv 0 \pmod m$, the observed model isgiven by:

$$y_\tau = x_\tau' \beta + u_\tau \quad \tau = 1, \ldots, [\frac{T}{m}] \tag{2.3}$$

$$u_\tau = \rho_* u_{\tau-1} + \epsilon_\tau^* \tag{2.4}$$

and ϵ_τ^* is i. i. d. N (0, σ_*^2) where $\sigma_*^2 = (1 + \rho^2 + \ldots + \rho^{2(m-1)})\sigma^2$ and $\rho_* = \rho^m$.

Proof: see Appendix I

In order to obtain the log likelihood function for the model (2.3-4) the Kalman Filter recursions will be used.

The state space representation of the model (2.3-4) is given by the measurement equation, i. e.:

$$y_\tau = (1, 0)\alpha_\tau \quad \tau = 1, \ldots, [\frac{T}{m}] \tag{2.5}$$

$$\Leftrightarrow y_\tau = Z_\tau \alpha_\tau$$

and the transition equation, i. e.:

$$\alpha_\tau = \begin{bmatrix} 0 & \rho_* \\ 0 & \rho_* \end{bmatrix} \alpha_{\tau-1} + \begin{bmatrix} x_\tau' \beta \\ 0 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} \epsilon_\tau^* \tag{2.6}$$

$$\Leftrightarrow \alpha_\tau = \Phi \alpha_{\tau-1} + c_\tau + R\epsilon_\tau^*$$

where $\mathrm{Var}(\epsilon_\tau^*) = \sigma_*^2 Q$.

The log likelihood function can be obatined using the prediction error decomposition (see Harvey (1981)) and is given by:

$$l(\Psi) = ln \ \ L(\rho_*, \beta', \sigma_*^2) = -\frac{T}{2m} ln \ \ 2\pi - \frac{T}{2m} ln \ \ \sigma_*^2$$

$$- \frac{1}{2} \sum_{\tau=1}^{[\frac{T}{m}]} ln \ \ f_\tau - \frac{1}{2\sigma_*^2} \sum_{\tau=1}^{[\frac{T}{m}]} \frac{\nu_\tau^2}{f_\tau} \tag{2.7}$$

where ν_τ is the prediction error and $\sigma_*^2 f_\tau$ is the variance of the prediction error and they are obtained by the Kalman Filter recursions (see Harvey (1981) pages 107 - 111). These recursions are given by two sets of equations: the prediction equations and the updating equations. In order to start these recursions, the mean, a_0, and the variance - covariance matrix, $\sigma_*^2 P_0$, of the state space vector at time $t = 0$ are needed. The log likelihood (2.7) is obtained when these staring values are given by:

$$a_0 = 0$$

$$P_0 = \begin{bmatrix} \kappa & 0 \\ 0 & \frac{1}{1-\rho_*^2} \end{bmatrix}$$

where κ is a large number, e. g. $\kappa = 10^6$, which is equivalent to use a diffuse prior for the initial condictions (see Ansley & Kohn (1985)).

It is easy to see that

$$\nu_1 = y_1 - x_1'\beta \tag{2.8}$$

$$\nu_\tau = (y_\tau - x_\tau'\beta) - \rho_*(y_{\tau-1} - x_{\tau-1}'\beta) \quad \tau = 2, \ldots, [\frac{T}{m}] \tag{2.9}$$

and

$$f_1 = \frac{1}{1-\rho_*^2} \tag{2.10}$$

$$f_\tau = 1 \quad \tau = 2, \ldots, [\frac{T}{m}] \tag{2.11}$$

In order to derive the LM statsitic for testing for first order autocorrelation, the first derivatives of the log likelihood function and the information matrix are needed. Pagan (1978) or Engle & Watson (1981) have shown that the informattion matrix can be computed using only first derivatives of the likelihood function. These derivatives are computed using auxiliary recursions (see Appendix II).

By Breusch & Pagan (1978), the LM statistic for testing for first order autocorrelation is given by:

$$LM = \widetilde{D}_1 \left(\widetilde{I}_{\rho_*\rho_*} - \widetilde{I}_{\rho_*\beta}\widetilde{I}_{\beta\beta}^{-1}\widetilde{I}_{\beta\rho_*}\right)^{-1} \widetilde{D}_1 \tag{2.12}$$

where

$$\widetilde{D} = \begin{bmatrix} \frac{\partial l}{\partial \rho_*} \\ \frac{\partial l}{\partial \beta} \\ \frac{\partial l}{\partial \sigma_*^2} \end{bmatrix} (\widetilde{\Psi}) = \begin{bmatrix} \widetilde{D}_1 \\ 0 \\ 0 \end{bmatrix}$$

and

$$\widetilde{I} = \begin{bmatrix} I_{\rho_*\rho_*} & I_{\rho_*\beta} & I_{\rho_*\sigma_*^2} \\ I_{\beta\rho_*} & I_{\beta\beta} & I_{\beta\sigma_*^2} \\ I_{\sigma_*^2\rho_*} & I_{\sigma_*^2\beta} & I_{\sigma_*^2\sigma_*^2} \end{bmatrix} (\widetilde{\Psi})$$

is the Fisher information matrix evaluated at $\widetilde{\Psi} = \Psi_{H_0} = (0, \widetilde{\beta}, \widetilde{\sigma}_*^2)$ and

$$\widetilde{\beta} = \left[(X^*)'(X^*)\right]^{-1}(X^*)'Y^*$$
$$X^* = (x(m), x(2m), \ldots, x(T))$$
$$Y^* = (y(m), y(2m), \ldots, y(T))$$
$$\widetilde{\sigma}_*^2 = \frac{1}{\frac{T}{m}} \sum_{\tau=1}^{[\frac{T}{m}]} \left(y_\tau - x_\tau'\widetilde{\beta}\right)^2 = \frac{1}{\frac{T}{m}} \sum_{\tau=1}^{[\frac{T}{m}]} \widetilde{u}_\tau^2$$

From Appendix III it follows that:

$$LM = \frac{T}{m}(r_1^*)^2 \tag{2.13}$$

where

$$r_1^* = \frac{\sum_{\tau=1}^{\left[\frac{T}{m}\right]} \tilde{u}_\tau \tilde{u}_{\tau-1}}{\sum_{\tau=1}^{\left[\frac{T}{m}\right]} \tilde{u}_\tau} \tag{2.14}$$

which is exactly the s_1 statistic derived by Robinson (1985).

3 THE LM STATISTICS FOR TESTING FIRST ORDER SERIAL CORRELATION IN TEMPORALLY AGGREGATED REGRESSION MODELS

Now it is assumed that the endogenous variable y(t) in (2.1) is a flow variable and instead of observing y(t) every time period, is y_τ which is observed, i. e.:

$$y_\tau = \sum_{j=0}^{m-1} y(m\tau - j) \quad \tau = 1, \ldots, \left[\frac{T}{m}\right] \tag{3.1}$$

Given that model (2.1-2) is the disaggregated model, i. e. for times $t = 1, \ldots, t$, Proposition II below derives the aggregated model, i. e. for $\tau = 1, \ldots, \left[\frac{T}{m}\right]$ and the endogenous variable is given by (3.1).

Proposition II.

Given model (2.1-2) and assume that y_τ is given by (3.1). Then the aggregated model is given by:

$$y_\tau = z_\tau' \beta + u_\tau \quad \tau = 1, \ldots, \left[\frac{T}{m}\right] \tag{3.2}$$

$$u_\tau = \rho_* u_{\tau-1} + \epsilon_\tau^* + \theta_* \epsilon_{\tau-1}^* \tag{3.3}$$

where

$$z_\tau' = \sum_{j=0}^{m-1} x'(m\tau - j)$$
$$\rho_* = \rho^m$$
$$\epsilon_\tau^* \sim N(0, \sigma_*^2)$$

and θ_* and σ_*^2 are given by the solutions of the following equations:

$$\begin{aligned}(1+\theta_*^2)\sigma_*^2 = {} & [1 + (1+\rho)^2 + (1+\rho+\rho^2)^2 + \ldots + (1+\rho+\ldots+\rho^{m-1})^2 + \\ & + \rho^2(1+\rho+\ldots+\rho^{m-2})^2 + \rho^4(1+\rho+\ldots+\rho^{m-3})^2 + \\ & + \ldots + \rho^{2(m-1)}]\sigma^2 \end{aligned} \tag{3.4}$$

$$\begin{aligned}\theta_*\sigma_*^2 = {} & [\rho(1+\rho+\ldots+\rho^{m-2}) + (1+\rho)\rho^2(1+\rho+\ldots+\rho^{m-3}) + \\ & + \ldots + (1+\rho+\ldots+\rho^{m-3})\rho^{m-2}(1+\rho) + \\ & + (1+\rho+\ldots+\rho^{m-2})\rho^{m-1}]\sigma^2 \end{aligned} \tag{3.5}$$

$$|\theta_*| < 1 \tag{3.6}$$

Proof: : see Appendix IV

The LM statistic for testing first order serial correlation will be derived using the Kalman Filter recursions. In order to use these recursions the state space representation of model (3.2-3) is needed.

This representation is given by the measurement equation, i. e.:

$$
\begin{aligned}
y_\tau &= (0,0,1)\,\alpha_\tau \quad \tau = 1,\ldots,\left[\frac{T}{m}\right] \\
\Leftrightarrow y_\tau &= Z_\tau \alpha_\tau
\end{aligned}
\tag{3.7}
$$

and the transition equation, i. e.:

$$
\begin{aligned}
\alpha_\tau &= \begin{bmatrix} \rho_\star & 1 & 0 \\ 0 & 0 & 0 \\ \rho_\star & 1 & 0 \end{bmatrix} \alpha_{\tau-1} + \begin{bmatrix} 0 \\ 0 \\ z_\tau' \beta \end{bmatrix} + \begin{bmatrix} 1 \\ \theta_\star \\ 1 \end{bmatrix} \epsilon_\tau^\star \\
\Leftrightarrow \alpha_\tau &= \Phi \alpha_{\tau-1} + c_\tau + R\epsilon_\tau^\star
\end{aligned}
\tag{3.8}
$$

where $\mathrm{Var}(\epsilon_\tau^\star) = \sigma_\star^2 Q$.

The Kalman Filter recursions are started by

$$
a_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
P_0 = \begin{bmatrix} \frac{1+2\rho_\star\theta_\star+\theta_\star^2}{1-\rho_\star^2} & \theta_\star & 0 \\ \theta_\star & \theta_\star^2 & 0 \\ 0 & 0 & \kappa \end{bmatrix}
$$

and the log likelihood function is given by (2.7).

It is easy to see that:

$$
\nu_1 = y_1 - z_1'\beta \tag{3.9}
$$

$$
\nu_\tau = (y_\tau - z_\tau'\beta) - \rho_\star(y_{\tau-1} - z_{\tau-1}'\beta) - \theta_\star f_{\tau-1}^{-1}\nu_{\tau-1} \quad \tau = 2,\ldots,[\frac{T}{m}] \tag{3.10}
$$

and

$$
f_1 = \frac{1+2\rho_\star\theta_\star+\theta_\star^2}{1-\rho_\star^2} \tag{3.11}
$$

$$
f_\tau = 1 + \theta_\star^2\left(1 - f_{\tau-1}^{-1}\right) \qquad \tau = 2,\ldots,[\frac{T}{m}] \tag{3.12}
$$

It is easy to see that under H_0 (and under some conditions on the x(t)), i. e. $\rho = 0 \Leftrightarrow \rho_\star = 0$:

(i) the OLS estimate of β, i. e. $\tilde{\beta}$ is given by:

$$
\tilde{\beta} = (Z'Z)^{-1}Z'Y^\star \tag{3.13}
$$

where $Z = (z_1, \ldots, z_{[\frac{T}{m}]})$, $z_\tau = \sum_{j=0}^{m-1} x(m\tau - j)$;

(ii) the prediction error is given $\nu_\tau = y_\tau - z_\tau' \tilde{\beta} \quad \forall \tau$;

(iii) the variance of the prediction error is given by $f_\tau = 1 \quad \forall \tau$;

(iv) and $\tilde{\sigma}_*^2 = m\tilde{\sigma}^2 = \frac{1}{\frac{T}{m}} \sum_{\tau=1}^{[\frac{T}{m}]} (y_\tau - z_\tau' \tilde{\beta})^2 = \frac{1}{\frac{T}{m}} \sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_*^2$.

From Appendix V, it follows that:

$$LM = \frac{T}{m} (r_1^*)^2 \tag{3.14}$$

where

$$r_1^* = \frac{\sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_\tau \tilde{u}_{\tau-1}}{\sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_\tau} \tag{3.15}$$

4 CONCLUDING REMARKS

It was shown that the LM statistic for testing first order serial correlation can be derived as a by - product of the Kalman Filter recursions.

In order to use the Kalman Filter recursions to derive the LM statistic a reparametrization was necessary.

It was shown that the LM statistic for testing first order serial correlation in regression models with missing observations, is equivalent to the s_1 statistic derived by Robinson (1985). This statistic is obtained from the likelihood conditional on the observation times. The LM statistic can be interpreted as $[\frac{T}{m}]$ times R^2, the coefficient of determination of the regression of $\tilde{u}_\tau$ in $\tilde{u}_{\tau-1}$, where $\tilde{u}_\tau = y_\tau - x_\tau'$, i. e. the residuals, under H_0, of model (2.1) for every m time period. An extension for regression models where the endogenous variable is observed at every time period for $t = 1, \ldots, p_0 T \quad (1 \le p_0 < 1)$ and for $t = p_0 T + 1, \ldots, T$ is observed at every m time period, is given in Pereira (1985).

Using the Kalman Filter recursions, it was possible to extend this result to temporally aggregated regression models. As before the LM statistic can be interpreted as $[\frac{T}{m}]$ times R^2 coefficient of determination of the regression of $\tilde{u}_\tau$ in $\tilde{u}_{\tau-1}$, where $\tilde{u}_\tau = y_\tau - z_\tau'$ and $z_\tau = \sum_{j=0}^{m-1} x(m\tau - j)$, i. e. the residuals under H_0 of model (3.2-3).

APPENDIX I

(2.1-2) can be rewritten as follows:

$$(1 - \rho L) y(t) = (1 - \rho L) x'(t) \beta + \epsilon(t) \tag{A.I.1}$$

Multiplying both sides of (A.I.1) by $(1 + \rho L + \ldots + \rho^{m-1} L^{m-1})$, it follows:

$$(1 - \rho^m L^m) y(t) = (1 - \rho^m L^m) x'(t) \beta + (1 + \rho L + \ldots + \rho^{m-1} L^{m-1}) \epsilon(t) \tag{A.I.2}$$

Now define

$$
\begin{aligned}
\mathcal{B} &= L^m \\
\rho_\star &= \rho^m \\
\epsilon_\tau^* &= (1+\rho L+\ldots+\rho^{m-1}L^{m-1})\epsilon(t) \quad \tau=1,\ldots,\left[\frac{T}{m}\right]
\end{aligned}
$$

then (A.I.2) can be rewritten as follows:

$$(1-\rho_\star\mathcal{B})y_\tau = (1-\rho_\star\mathcal{B})x_\tau'\beta+\epsilon_\tau^* \tag{A.I.3}$$

$$\Updownarrow$$

$$y_\tau = x_\tau'\beta + u_\tau \tag{A.I.4}$$

$$u_\tau = \rho_\star u_{\tau-1}+\epsilon_\tau \tag{A.I.5}$$

Using the results of Tiao(1972) or Pereira(1986) it follows that $\epsilon_\tau^* \sim NID(0,\sigma_\star^2)$ where $\sigma_\star^2 = (1+\rho^2+\ldots+\rho^{2(m-1)})\sigma^2$.

APPENDIX II

Taking derivatives of (2.3) with respect to the parameters, it follows that:

$$
\begin{aligned}
\frac{\partial l}{\partial\psi_i} &= \frac{1}{2\sigma_\star^2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{\nu_\tau^2}{f_\tau}\frac{\partial f_\tau}{\partial\psi_i} - \frac{1}{2\sigma_\star 2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{\nu_\tau}{f_\tau}\frac{\partial\nu_\tau}{\partial\psi_i} \\
&\quad -\frac{1}{2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{1}{f_\tau}\frac{\partial f_\tau}{\partial\psi_i} \quad i=1,\ldots,k+1
\end{aligned}
\tag{A.II.1}
$$

$$\frac{\partial l}{\partial\sigma_\star^2} = \frac{T}{2m\sigma_\star^2}+\frac{1}{2\sigma_\star 4}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{\nu_\tau^2}{f_\tau} \tag{A.II.2}$$

and the information matix is given by:

$$I_{\psi_i\psi_j} = \frac{1}{\sigma_*^2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{1}{f_\tau}\left[\frac{\partial \nu_\tau}{\partial \psi_i}\right]\left[\frac{\partial \nu_\tau}{\partial \psi_j}\right] - \frac{1}{2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{1}{f_\tau^2}\frac{\partial f_\tau}{\partial \psi_i}\frac{\partial f_\tau}{\partial \psi_j} \quad i,j = 1,\ldots,k+1 \tag{A.II.3}$$

$$I_{\sigma_*^2\psi_j} = \frac{1}{\sigma_*^4}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{\nu_\tau}{f_\tau}\frac{\partial \nu_\tau}{\partial \psi_i} - \frac{1}{2\sigma_*^4}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\frac{\nu_\tau^2}{f_\tau}\frac{\partial f_\tau}{\partial \psi_i} \quad i = 1,\ldots,k+1 \tag{A.II.4}$$

$$I_{\sigma_*^2\sigma_*^2} = \frac{T}{2m\sigma_*^4} \tag{A.II.5}$$

In order to obtain (A.II.1-4), it is needed auxiliarly recursions for the derivatives. They are:

Prediction Equations

$$\frac{\partial a_{\tau/\tau-1}}{\partial \psi_i} = \frac{\partial \Phi}{\partial \psi_i}a_{\tau-1} + \Phi\frac{\partial a_{\tau-1}}{\partial \psi_i} + \frac{\partial c_\tau}{\partial \psi_i} \tag{A.II.6}$$

$$\frac{\partial P_{\tau/\tau-1}}{\partial \psi_i} = \frac{\partial \Phi}{\partial \psi_i}P_{\tau-1}\Phi' + \Phi\frac{\partial P_{\tau-1}}{\partial \psi_i}\Phi' + \Phi P_{\tau-1}\left(\frac{\partial \Phi}{\partial \psi_i}\right)' + \frac{\partial R}{\partial \psi_i}QR' + R\frac{\partial Q}{\partial \psi_i}R' + RQ\left(\frac{\partial R}{\partial \psi_i}\right)' \quad i = 1,\ldots,k+1 \tag{A.II.7}$$

Prediction Error

$$\frac{\partial f_\tau}{\partial \psi_i} = Z_\tau'\frac{\partial P_{\tau/\tau-1}}{\partial \psi_i}Z_\tau \quad i = 1,\ldots,k+1 \tag{A.II.8}$$

Variance of the Prediction Error

$$\frac{\partial \nu_\tau}{\partial \psi_i} = -Z_\tau'\frac{\partial a_{\tau/\tau-1}}{\partial \psi_i} \quad i = 1,\ldots,k+1 \tag{A.II.9}$$

Updating Equations

$$
\begin{aligned}
\frac{\partial a_\tau}{\partial \psi_i} &= \frac{\partial a_{\tau/\tau-1}}{\partial \psi_i} + \frac{\partial P_{\tau/\tau-1}}{\partial \psi_i} Z_\tau f_\tau^{-1} \nu_\tau \\
&\quad - P_{\tau/\tau-1} Z_\tau f_\tau^{-2} \frac{\partial f_\tau}{\partial \psi_i} \nu_\tau + P_{\tau/\tau-1} Z_\tau f_\tau^{-1} \frac{\partial \nu_\tau}{\partial \psi_i} \quad i = 1, \ldots, k+1 \qquad \text{(A.II.10)} \\
\frac{\partial P_\tau}{\partial \psi_i} &= \frac{\partial P_{\tau/\tau-1}}{\partial \psi_i} - \frac{\partial P_{\tau/\tau-1}}{\partial \psi_i} Z_\tau f_\tau^{-1} Z_\tau' P_{\tau/\tau-1} \\
&\quad + P_{\tau/\tau-1} Z_\tau f_\tau^{-2} \frac{\partial f_\tau}{\partial \psi_i} Z_\tau' P_{\tau/\tau-1} \\
&\quad - P_{\tau/\tau-1} Z_\tau f_\tau^{-1} Z_\tau' \frac{\partial P_{\tau/\tau-1}}{\partial \psi_i} \quad i = 1, \ldots, k+1 \qquad \text{(A.II.11)}
\end{aligned}
$$

and the initial conditions are given by:

$$
\begin{aligned}
&\frac{\partial a_0}{\partial \psi_i} = 0 \quad i = 1, \ldots, k+1 \\
&\frac{\partial P_0}{\partial \beta} = 0 \\
&\frac{\partial P_0}{\partial \rho_*} = \begin{bmatrix} 0 & 0 \\ 0 & \frac{-2\rho_*}{(1-\rho_*^2)^2} \end{bmatrix} \\
&\frac{\partial P_0}{\partial \rho_*} = \begin{bmatrix} \frac{2(\theta_* + (\theta_* + \rho_*)\left[\frac{\partial \theta_*}{\partial \rho_*}\right])(1-\rho_*^2) + 2\rho_*(1+2\rho_*\theta_* + \theta_*^2)}{(1-\rho_*^2)^2} & \frac{\partial \theta_*}{\partial \rho_*} & 0 \\ \frac{\partial \theta_*}{\partial \rho_*} & 2\theta_* \frac{\partial \theta_*}{\partial \rho_*} & 0 \\ 0 & 0 & 0 \end{bmatrix}
\end{aligned}
$$

APPENDIX III

It is easy to see that

$$
\frac{\partial \nu_\tau}{\partial \psi_i} = \left[\frac{\partial \nu_\tau}{\partial \rho_*}, \frac{\partial \nu_\tau}{\partial \beta}, 0\right] = \begin{cases} (0, -x_1', 0) & \tau = 1 \qquad \text{(A.III.1)} \\ -(y_{\tau-1} - x_{\tau-1}'\beta, x_\tau' - \rho_* x_{\tau-1}', 0) & \tau > 1 \qquad \text{(A.III.2)} \end{cases}
$$

$$
\frac{\partial f_\tau}{\partial \psi_i} = \left[\frac{\partial f_\tau}{\partial \rho_*}, \frac{\partial f_\tau}{\partial \beta}, 0\right] = \begin{cases} (\frac{-2\rho_*}{1-\rho_*^2}, 0, 0) & \tau = 1 \qquad \text{(A.III.3)} \\ 0 & \tau > 1 \qquad \text{(A.III.4)} \end{cases}
$$

Substituting (A.III.1-4) and (2.8-11) into (A.II.1), it follows that:

$$\frac{\partial l}{\partial \rho_*} = \frac{-\rho_*(y_1 - x_1'\beta)^2}{\sigma_*^2} + \frac{1}{\sigma_*^2}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}\left[(y_\tau - x_\tau'\beta) - \rho_*(y_{\tau-1} - x_{\tau-1}'\beta)\right](y_{\tau-1} - x_{\tau-1}'\beta) + \frac{\rho_*}{1-\rho_*^2} \tag{A.III.5}$$

$$\frac{\partial l}{\partial \beta} = \frac{1-\rho_*^2}{\sigma_*^2}x_1'(y_1 - x_1'\beta) + \frac{1}{\sigma_*^2}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}(x_\tau - \rho_* x_\tau')'\left[(y_\tau - x_\tau'\beta) - \rho_*(y_{\tau-1} - x_{\tau-1}'\beta)\right] \tag{A.III.6}$$

Substituting (2.8-11) into (A.II.2), it follows that:

$$\frac{\partial l}{\partial \sigma_*^2} = -\frac{T}{2m\sigma_*^2} + \frac{1-\rho_*^2}{2\sigma_*^4}x_1'(y_1 - x_1'\beta)^2 + \frac{1}{\sigma_*^4}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}\left[(y_\tau - x_\tau'\beta) - \rho_*(y_{\tau-1} - x_{\tau-1}'\beta)\right]^2$$

Substituting (2.8-11) and (A.III.1-4) into (A.II.3), it follows that:

$$I_{\rho_*\rho_*} = \frac{1}{\sigma_* 2}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}(y_{\tau-1} - x_{\tau-1}'\beta)^2 + \frac{2\rho_*^2}{1-\rho_*^2} \tag{A.III.7}$$

$$I_{\rho_*\beta} = \frac{1}{\sigma_*^2}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}(x_\tau - \rho_* x_\tau')'(y_{\tau-1} - x_{\tau-1}'\beta) \tag{A.III.8}$$

$$I_{\beta\beta} = \frac{1}{\sigma_*^2}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}(x_\tau - \rho_* x_\tau')(x_\tau - \rho_* x_{\tau-1})' + \frac{1-\rho_*^2}{\sigma_*^2}(x_1 x_1') \tag{A.III.9}$$

Sunstituting (A.III.1-4) and (2.8-11) into (A.II.4), it follows that:

$$I_{\rho_*\sigma_*^2} = \frac{-\rho_*}{\sigma_*^4}(y_1 - x_1'\beta) - \frac{1}{\sigma_*^4}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}\left[(y_\tau - x_\tau'\beta) - \rho_*(y_{\tau-1} - x_{\tau-1}'\beta)\right](y_{\tau-1} - x_{\tau-1}'\beta) \quad \text{(A.III.10)}$$

$$I_{\beta\sigma_*^2} = \frac{1-\rho_*^2}{\sigma_*^4}x_1'(y_1 - x_1'\beta) - \frac{1}{\sigma_*^4}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}(x_\tau - \rho_* x_\tau')'\left[(y_\tau - x_\tau'\beta) - \rho_*(y_{\tau-1} - x_{\tau-1}'\beta)\right] \quad \text{(A.III.11)}$$

Under $H_0 : \rho = 0 \Leftrightarrow H_0; \rho_* = 0$, it follows that:

$$\frac{\partial l}{\partial \beta}(\psi_{H_0}) = 0$$
$$\frac{\partial l}{\partial \sigma_*^2}(\psi_{H_0}) = 0$$
$$\frac{\partial l}{\partial \rho_*}(\psi_{H_0}) = \frac{1}{\tilde{\sigma}_*^2}\sum_{\tau=2}^{\left[\frac{T}{m}\right]}\tilde{u}_\tau\tilde{u}_{\tau-1} \quad \text{(A.III.12)}$$

Taking plim' s in (A.III.7-11), and under appropriated regularity conditions on the $x'(t)$, it follows that:

$$plim\left[\tilde{I}_{\rho_*\rho_*} - \frac{1}{\tilde{\sigma}_*^2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\tilde{u}_{\tau-1}^2\right] = 0$$
$$plim\left[\tilde{I}_{\beta\beta} - \frac{1}{\tilde{\sigma}_*^2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}x_\tau x_\tau'\right] = 0$$
$$plim \tilde{I}_{\rho_*\beta} = 0$$
$$plim \tilde{I}_{\rho_*\beta} = 0$$
$$plim \tilde{I}_{\beta\sigma_*^2} = 0$$

It follows that

$$LM = \left[\frac{1}{\tilde{\sigma}_*^2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\tilde{u}_\tau\tilde{u}_{\tau-1}\right]\left[\frac{1}{\tilde{\sigma}_*^2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\tilde{u}_{\tau-1}^2\right]^{-1}\left[\frac{1}{\tilde{\sigma}_*^2}\sum_{\tau=1}^{\left[\frac{T}{m}\right]}\tilde{u}_\tau\tilde{u}_{\tau-1}\right]$$

but

$$\left[\frac{1}{\tilde{\sigma}_*^2}\sum_{\tau=1}^{[\frac{T}{m}]}\tilde{u}_{\tau-1}^2\right] = \frac{T}{m}$$

then

$$LM = \left[\frac{T}{m}\right]^2 r_1^{*^2}\left[\frac{T}{m}\right]^{-1} = \left[\frac{T}{m}\right] r_1^{*^2}$$

where

$$r_1^{*^2} = \frac{\sum_{\tau=1}^{[\frac{T}{m}]}\tilde{u}_\tau\tilde{u}_{\tau-1}}{\sum_{\tau=1}^{[\frac{T}{m}]}\tilde{u}_\tau^2}$$

APPENDIX IV

Proof of Proposition II:

Multiplying both sides of (A.I.2) by $\sum_{j=1}^{m-1} L^j$, it follows that:

$$(1-\rho^m B)y_\tau = (1-\rho^m)\left(\sum_{j=0}^{m-1} x'(t-j)\right)\beta + \omega_\tau \qquad \text{(A.IV.1)}$$

where

$$\omega_\tau = \left(\sum_{j=0}^{m-1}\right)\left(\sum_{j=1}^{m-1}\rho^j L^j\right)\epsilon(t) \qquad \text{(A.IV.2)}$$

Now (A.IV.1) can be rewritten as:

$$y_\tau = z_\tau'\beta + u_\tau \qquad \text{(A.IV.3)}$$
$$u_\tau = \rho_* u_{\tau-1} + \omega_\tau \qquad \text{(A.IV.4)}$$

where

$$z_\tau' = \sum_{j=0}^{m-1} x(m\tau - j)$$
$$\rho_* = \rho^m$$

and ω_τ is given by (A.IV.2).

Using the results of Tiao (1972) or Pereira (1986), (A.IV.4) follows an ARMA(1,1), the expression (3.3), where the AR parameter is $\rho_\star$ and the MA parameter, $\theta_\star$ and the variance of the error term, $\sigma_\star^2$, are given by the solution of equation (3.4-6).

APPENDIX V

Derivating (3.9-12) with respect to β, it follows that:

$$\frac{\partial \nu_\tau}{\partial \beta} = \begin{cases} -z_1^t & \text{(A.V.1)} \\ -[z_\tau - \rho_\star z_{\tau-1}] + \theta_\star \frac{\partial f_{\tau-1}}{\partial \beta} f_{\tau-1}^{-2} \nu_{\tau-1} - \theta_\star f_{\tau-1}^{-1} \frac{\partial \nu_{\tau-1}}{\partial \beta} & \tau > 1 \text{ (A.V.2)} \end{cases}$$

and

$$\frac{\partial f_\tau}{\partial \beta} = 0 \tag{A.V.3}$$

Now using (A.V.3) into (A.V.2), it follows that:

$$\frac{\partial \nu_\tau}{\partial \beta} = \begin{cases} -z_1^t & \text{(A.V.1)} \\ -[z_\tau - \rho_\star z_{\tau-1}] - \theta_\star f_{\tau-1}^{-1} \frac{\partial \nu_{\tau-1}}{\partial \beta} & \tau > 1 \quad \text{(A.V.2)} \end{cases}$$

Now derivating (3.9-12) with respect to $\rho_\star$, it follows that:

$$\frac{\partial \nu_\tau}{\partial \rho_\star} = \begin{cases} 0 & \tau = 1 \text{ (A.V.4)} \\ -\left[y_{\tau-1} - z_{\tau-1}^t \beta\right] - \frac{\partial \theta_\star}{\partial \rho_\star} f_{\tau-1}^{-1} \nu_{\tau-1} & \\ +\theta_\star \frac{\partial f_{\tau-1}}{\partial \rho_\star} f_{\tau-1}^{-2} \nu_{\tau-1} - \theta_\star f_{\tau-1}^{-1} \frac{\partial \nu_{\tau-1}}{\partial \rho_\star} & \tau > 1 \text{ (A.V.5)} \end{cases}$$

and

$$\frac{\partial f_\tau}{\partial \rho_\star} = \begin{cases} \frac{\left[2\theta_\star + 2\rho_\star \frac{\partial \theta_\star}{\partial \rho_\star} + 2\theta_\star \frac{\partial \theta_\star}{\partial \rho_\star}\right](1-\rho_\star^2) - 2\rho_\star\left[1 + 2\rho_\star\theta_\star + \theta_\star^2\right]}{(1-\rho_\star^2)^2} & \tau = 1 \text{ (A.V.6)} \\ 2\theta_\star \frac{\partial \theta}{\partial \rho_\star}(1 - f_{\tau-1}^{-1}) + \theta_\star^2 \frac{\partial f_{\tau-1}}{\partial \rho_\star} f_{\tau-1}^{-2} & \tau > 1 \text{ (A.V.7)} \end{cases}$$

Derivating (3.9-12) with respect to $\sigma_\star^2$, it follows that:

$$\frac{\partial f_\tau}{\partial \sigma_\star^2} = \frac{\partial \nu_\tau}{\partial \sigma_\star^2} = 0$$

Then under H_0, it follows that:

$$\frac{\partial \nu_\tau}{\partial \rho_\star}(\psi_{H_0}) = \begin{cases} 0 & \tau = 1 \text{ (A.V.8)} \\ -\frac{m+1}{m}\left[y_{\tau-1} - z_{\tau-1}^t \tilde{\beta}\right] & \tau > 1 \text{ (A.V.9)} \end{cases}$$

$$\frac{\partial f_\tau}{\partial \rho_*}(\psi_{H_0}) = 0 \quad \forall \tau \tag{A.V.10}$$

$$\frac{\partial \nu_\tau}{\partial \beta}(\psi_{H_0}) = \begin{cases} -z_1 & \tau = 1 \text{ (A.V.11)} \\ -z_\tau & \tau > 1 \text{ (A.V.12)} \end{cases}$$

$$\frac{\partial f_\tau}{\partial \beta}(\psi_{H_0}) = 0 \quad \forall \tau \tag{A.V.13}$$

Substituting (A.V.8-13) into (A.II.1), it follows that:

$$\begin{aligned} &\frac{\partial l}{\partial \beta}(\psi_{H_0}) = 0 \\ &\frac{\partial l}{\partial \sigma_*^2}(\psi_{H_0}) = 0 \\ &\frac{\partial l}{\partial \rho_*}(\psi_{H_0}) = \frac{m+1}{m\tilde{\sigma}_*^2} \sum_{\tau=2}^{[\frac{T}{m}]} \tilde{u}_\tau \tilde{u}_{\tau-1} \end{aligned} \tag{A.V.14}$$

Taking plim' s in the expressions fot the information matrix and under appropriated regularity conditions on the $x'(t)$, it follows that:

$$\begin{aligned} plim\left[\tilde{I}_{\rho_*\rho_*} - \left[\frac{m+1}{m}\right]^2 \frac{1}{\tilde{\sigma}_*^2} \sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_{\tau-1}^2\right] &= 0 \\ plim\left[\tilde{I}_{\beta\beta} - \frac{1}{\tilde{\sigma}_*^2} \sum_{\tau=1}^{[\frac{T}{m}]} z_\tau z_\tau'\right] &= 0 \\ plim \tilde{I}_{\rho_*\beta} &= 0 \\ plim \tilde{I}_{\rho_*\beta} &= 0 \\ plim \tilde{I}_{\beta\sigma_*^2} &= 0 \end{aligned}$$

It follows that

$$LM = \left[\left[\frac{m+1}{m}\right] \frac{1}{\tilde{\sigma}_*^2} \sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_\tau \tilde{u}_{\tau-1}\right] \left[\left[\frac{m+1}{m}\right]^2 \frac{1}{\tilde{\sigma}_*^2} \sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_{\tau-1}^2\right]^{-1} \left[\left[\frac{m+1}{m}\right] \frac{1}{\tilde{\sigma}_*^2} \sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_\tau \tilde{u}_{\tau-1}\right]$$

but

$$\left[\frac{1}{\tilde{\sigma}_*^2} \sum_{\tau=1}^{[\frac{T}{m}]} \tilde{u}_{\tau-1}^2\right] = \frac{T}{m}$$

then

$$LM = \left[\frac{T}{m}\right]^2 r_1^{*^2} \left[\frac{T}{m}\right]^{-1} = \left[\frac{T}{m}\right] r_1^{*^2}$$

where

$$r_1^{*^2} = \frac{\sum_{\tau=1}^{\left[\frac{T}{m}\right]} \tilde{u}_\tau \tilde{u}_{\tau-1}}{\sum_{\tau=1}^{\left[\frac{T}{m}\right]} \tilde{u}_\tau^2}$$

and $\tilde{u}_\tau = y_\tau - z_\tau^t \tilde{\beta}$ and $\tilde{\beta}$ is given by (3.3).

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